Multiperiod Multiproduct Advertising Budgeting: Stochastic Optimization Modeling

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1.1 ABSTRACT

- We propose and analyze an **stochastic** model for the **Multiperiod Multiproduct Advertising Budgeting** problem.
- This model optimizes the **advertising investment:**
  - for several products,
  - by considering cross elasticities,
  - different sales **drivers** (advertising, promotions, media selection, timing, etc.)
  - for a multiperiod planning horizon.
- This model was tested to plan a realistic **advertising campaign.**
1.1. ABSTRACT

- The **stochastic** approach increased by **4.38%** the optimal expected profit, compared to the deterministic approach.

- The **deterministic** approach overestimated the corresponding expected profit by **38%**.
1.2 INTRODUCTION

- We address the Multiperiod Multiproduct Advertising Budgeting problem (for short: \textbf{MAB} problem).
  
  - By \textbf{advertising budgeting} we mean that we wish to decide the capital to invest on advertising and how to allocate it.
  
  - By \textbf{multiproduct} we mean that we simultaneously optimize:
    - the advertising campaigns of different products within the same company,
    - by considering \textbf{cross product effects}.
Our approach is **multichannel** since we consider different media (television, radio, internet, etc.).

By **multiperiod** we mean that the planning periods are optimized simultaneously.

- Every year many companies spend **thousands of euros** to advertise and promote their products.
- Appropriate forecasting and **optimizing technologies** can help:
  - either to obtain better **advertising results** for a given budget,
  - or to reduce the **advertising expenses**.
• A stochastic model of the MAB problem has not been addressed in literature.

• Relevant profit level and profit accuracy can be improved by using a stochastic model instead of a deterministic one.

• The main contribution of this presentation is to propose and analyze a stochastic model for the MAB problem.

• The model we propose is convex and can be solved by standard optimization software.
1.3 PROBLEM FORMULATION (PF)

- **GRP** stands for ‘Gross Rating Point’. For example 100 GRPs means that 100% of the market is exposed once to an advertising, or that 50% of the market is exposed twice.

- Usually, investment in advertising is measured in **GRPs** and not in currency units (euro, dollar, etc.).

- We distinguish between baseline sales (sales that one would expect without advertising) and **sales due to advertising**.

- The objective of the MAB problem is to **maximize the profit** of the sales due to advertising.
Advertising Profit = Profit$_u \times$ Sales$_g -$ Cost$_g$

- Profit$_u$ is the profit per unit sold,
- Sales$_g$ corresponds to the sales due to advertising which is a function of $g$, the number of GRPs,
- Cost$_g$ is the cost of buying $g$ GRPs.

- Notice that in the whole presentation the decision vector is $g$

the investment on advertising (in GRPs).
1.4 PF: THE SALES RESPONSE FUNCTION

- To model Sales\(_g\), (statistical) **response models** can be constructed.

- **Sales response functions** \( S(g) \) correspond to increasing concave functions which model diminishing returns.

- A typical choice, among others, is the so called ‘**modified exponential**’ function

\[
S(g) = \alpha \left(1 - e^{-\beta g}\right).
\]

- The **cross advertising effect** \( i,j \) corresponds to the sales induced on product \( i \) by advertising product \( j \).
• We approximate the cross advertising effect $i j$ by a linear function:

$$C_{ij}(g_j) = \gamma_{ij} g_j.$$
1.5 PF: PROFIT FUNCTION

\[ P(g, \xi) = \sum_{tjk} P_{tjk}(g, \xi) \quad (1.1) \]

\[ t, j, k = \text{period, product, driver} \quad (1.2) \]

\[ g = (g_{tjk}) \quad (1.3) \]

\[ \xi = (\alpha_{tjk}, \beta_{tjk}, \gamma_{tijk}, \varepsilon_{tjk}) \quad (1.4) \]

\[ P_{tjk}(g, \xi) = p_{tj} S_{tjk}(g) + \sum_{i \in I} p_{ti} C_{tijk}(g) - c_{tjk} g_{tjk} + \varepsilon_{tjk} \quad (1.5) \]

\[ S_{tjk}(g) = \alpha_{tjk} (1 - e^{-\beta_{tjk} x_{tjk}(g)}) \quad (1.6) \]

\[ C_{tijk}(g) = \gamma_{tijk} x_{tjk}(g) \quad (1.7) \]

\[ x_{tjk}(g) = \delta_{jk} x_{t-1,jk}(g) + g_{tjk} \quad \text{adstock variable} \quad (1.8) \]
1.6 PF: THE STOCHASTIC MODEL

• The stochastic model $\text{MAB}_S$, is defined as

$$\min_{g \in \mathcal{D}} F_S(g) := \mathbb{E}[-P(g, \xi)]$$

where

$$\mathcal{D} = \left\{ g \in \mathbb{R}^n \mid \sum_{tjk} c_{tjk} g_{tjk} \leq b, \quad g \leq \underline{g} \leq \overline{g} \right\}.$$

**Proposition 1** If each $(\alpha_{tjk}, \beta_{tjk})$, for all $tjk \in \mathcal{T\mathcal{J}\mathcal{K}}$, is a pair of
independent random variables, then

\[ \mathbb{E}[P(g, \xi)] = \sum_{t,j,k} \left\{ p_{tj} \, \tilde{\alpha}_{tjk} \left( 1 - \mathbb{E} \left[ e^{-\beta_{tjk} \, x_{tjk}(g)} \right] \right) \right. \]

\[ + \sum_{i} p_{ti} \, \tilde{\gamma}_{tijk} \, x_{tjk}(g) - c_{tjk} \, g_{tjk} \left\} , \]

where \( \tilde{\alpha}_{tjk} \) and \( \tilde{\gamma}_{tjk} \) represent expected values.

**Proposition 2** Model \( \text{MAB}_S \) is a **convex** optimization problem.
Figure 1.1: **Sample distribution** of $\beta$. The probability of each possible realization $\tilde{\beta}$ is given by $f_\beta(\tilde{\beta})$.

### 1.7 WHERE IS THE STOCHASTICITY?

- It is in the $\beta$’s.
1.7. WHERE IS THE STOCHASTICITY?

To compute $\mathbb{E} \left[ e^{-\beta g} \right]$ we have several cases:

1. There is an **analytical formula**. For example, if $\beta \sim N(\mu, \sigma^2)$ then:

$$\mathbb{E} \left[ e^{-\beta g} \right] = e^{-\mu g} + 0.5 \sigma^2 g^2.$$

2. There is not an analytical formula.

   a. If we have the **sample distribution** of $\beta$ as in Fig. 1.1 then we can compute:

   $$\mathbb{E} \left[ e^{-\beta g} \right] = \sum_{\tilde{\beta} \in \Omega} f(\tilde{\beta}) e^{-\tilde{\beta} g}, \quad (1.9)$$

   where $\{\tilde{\beta}^\omega\}$ are the possible scenarios and $f(\tilde{\beta})$ the weights.
b. We can make the following (crude?) approximation.

\[ \mathbb{E} \left[ e^{-\beta g} \right] \approx e^{-\mathbb{E}[\beta] g}. \]
1.8 PF: THE DETERMINISTIC MODEL

- The (deterministic) expected value model $\text{MAB}_{EV}$, is defined as

$$\min_{g \in \mathcal{D}} F_{EV}(g) := -P(g, \mathbb{E}[\xi]).$$

- Under the assumption of parameter independence model $\text{MAB}_{EV}$ is identical to model $\text{MAB}_S$ except the following (crude?) approximation:

$$\mathbb{E} \left[ e^{-\beta g} \right] \approx e^{-\mathbb{E}[\beta]} g.$$

Proposition 3 Model $\text{MAB}_{EV}$ is a convex optimization problem.
Proposition 4 If each \((\alpha_{tjk}, \beta_{tjk})\), for all \(tjk \in \mathcal{TJK}\), is a pair of independent random variables then the deterministic optimal solution \(g_{EV}^*\) gives a lower and an upper bound

\[ F_{EV}(g_{EV}^*) \leq F_S^* \leq F_S(g_{EV}^*). \]
1.9 CASE STUDY (CS)

- In this section we will try to answer the following questions:
  - Which is the **optimal advertising budget** for the whole planning horizon?
  - Given the optimal budget, which is the **optimal allocation** along the planning horizon?
  - Is it important to consider **stochastic** models? or on the contrary, is it enough to consider **deterministic** ones?

- The instance we present here:
  - considers a **twelve months** planning horizon.
  - **two products** that we denoted by P1 and P2.
1.9. CASE STUDY (CS)

- **two drivers** per product.
  - the first driver corresponds to **TV advertising**, 
  - the second driver corresponds to **in-store promotions**.

- The distribution of each $\beta$ is estimated by a **sample distribution** (see the following figure).
Figure 1.2: **Sample distribution** of $\tilde{\beta}_{11}$. The probability of each possible realization $\tilde{\beta}_{11}$ is given by $f_{\beta_{11}}(\beta_{11})$. 
1.10 CS: THE ‘MODIFIED EXPONENTIAL’ RESPONSE FUNCTION

- We use the single product sales response function denominated modified exponential:

  \[ S(g) = \alpha (1 - e^{-\beta g}), \]

  \( \alpha \) corresponds to the saturation level,

  \( \beta \) regulates the diminishing return.
1.11 CS: CROSS PRODUCT SALES EFFECTS

- The cross product sales effects between products P1 and P2 are due to substitution:
  - Advertising on, say P1, will increase P1 sales but will reduce P2 sales and vice versa.
  - These effects are known as cannibalization.
- Under cannibalization, the cross product effect parameter $\gamma_{21}$ is negative:
  $$C_{21}(g_1) = \gamma_{21} g_1.$$
Figure 1.3: Unidimensional **profit function** of advertising product P1, \( P_1(g_1) \) based on the modified exponential sales response function:

\[
P_1(g_1) = p_1 \alpha (1 - e^{-\beta g_1}) + p_2 \gamma_{21} g_1 - c_1 g_1.
\]
1.12 CS: DETERMINING THE OPTIMAL BUDGET

- When solving the MAB problem one can compute the optimal budget considering the uncertainty of the parameters (stochastic optimization).

- Alternatively, one can ignore the uncertainty of the parameters (deterministic optimization).

- The deterministic optimization may give a good solution but suboptimal for the stochastic problem.
1.12. CS: DETERMINING THE OPTIMAL BUDGET

Table 1.1: **Optimal expected profit** (euros): The expected profit given by the stochastic approach is **4.38% better** (738,146 euros) than the expected profit given by the deterministic approach.

<table>
<thead>
<tr>
<th></th>
<th>MAB&lt;sub&gt;EV&lt;/sub&gt;</th>
<th>EEV</th>
<th>MAB&lt;sub&gt;S&lt;/sub&gt;</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget</td>
<td>3,818,334</td>
<td>-</td>
<td>4,350,039</td>
<td>+13.93%</td>
</tr>
<tr>
<td>Deterministic profit</td>
<td>23,276,709</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Expected profit</td>
<td>-</td>
<td>16,870,731</td>
<td>17,608,877</td>
<td>+4.38%</td>
</tr>
<tr>
<td>CPU time (s)</td>
<td>2</td>
<td>-</td>
<td>54</td>
<td>-</td>
</tr>
<tr>
<td>SQP iterations</td>
<td>43</td>
<td>-</td>
<td>35</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 1.4: Optimal budget allocation $g_t^*$ for driver 1 of product P1: Deterministic approach versus stochastic approach.
1.13 CS: SIMULATION

• So far we have compared the deterministic versus the stochastic expected profit:

\[ \mathbb{E}[P(g_{EV}^*, \xi)] \text{ vs } \mathbb{E}[P(g_S^*, \xi)] \]

\[ 16,870,731 \text{ vs } 17,608,877 \]

• It can also be useful to compare the corresponding ‘optimal’ profits \( P(g_S^*, \xi) \) and \( P(g_{EV}^*, \xi) \), as random variables.

• For this task, we used a random sample of 20,000 scenarios (vectors) \( \{\tilde{\xi}^k\} \).

• For each scenario \( \tilde{\xi}^k \), we computed the corresponding profits \( P(g_S^*, \tilde{\xi}^k) \) and \( P(g_{EV}^*, \tilde{\xi}^k) \) to obtain the following results:
1.13. CS: SIMULATION

Table 1.2: **Sample optimal profit** (euros) for the deterministic and stochastic approaches.

<table>
<thead>
<tr>
<th></th>
<th>Deterministic</th>
<th>Stochastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>11,814,360</td>
<td>12,483,785</td>
</tr>
<tr>
<td>Max</td>
<td>21,071,046</td>
<td>21,242,252</td>
</tr>
<tr>
<td>Mean</td>
<td>16,855,996</td>
<td>17,619,252</td>
</tr>
<tr>
<td>Median</td>
<td>16,895,003</td>
<td>17,678,295</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1,305,596</td>
<td>1,093,411</td>
</tr>
</tbody>
</table>
Figure 1.5: Sample probability distribution of the profit (euros) that may be obtained with the deterministic approach (sample expected profit 16,855,996 euros).
1.13. CS: SIMULATION

Figure 1.6: Sample probability distribution of the profit (euros) that may be obtained with the stochastic approach (sample expected profit 17,619,252 euros).
Figure 1.7: Sample **cumulated probability**: It is more likely to obtain a low profit with the deterministic approach.
1.14 CONCLUDING REMARKS

• From a theoretical point of view:
  ▶ We have shown that the stochastic MAB model that we propose corresponds to a convex optimization problem:
    ◦ numerically tractable,
    ◦ produces global optimal solutions.

• From a practical point of view:
  ▶ We propose for the first time a stochastic multiperiod multiproduct model for the advertising budgeting problem.
  ▶ In our case study we have observed that:
1.14. CONCLUDING REMARKS

- The stochastic model has **increased by 4.38% the profit** of the deterministic model.
- The **deterministic** optimal profit overestimated the corresponding expected profit by 38%.

- Some **Limitations** of the model:
  - It does not take into account the **competitive aspects** of the problem.
  - It does not optimize the **prices** (they are input data).
Thank You!